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NUMERICAL ALGORITHMS FOR THE SOLUTION OF KINEMATIC PROBLEM AND DYNAMICAL CONTROL OF MANIPULATOR

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Abstract: *An industrial robot may be modeled by a series of links interconnected by either rotary or sliding joints driven by actuators. From a given angular configuration and the geometric manipulator parameters, it is possible to know the position and orientation of its end effector with respect to a coordinate system fixed in the base of the manipulator (the kinematic model is expressed in Cartesian coordinates). Industrial applications demand the robot to operate in accordance to the position and orientation of its end effector and it is necessary to solve the kinematic inverse problem to determine the joint displacement necessary to the movement of the joint of the manipulator to achieve a given objective. This paper focus the implementation of numerical algorithms for the solution of the kinematic inverse problem that may be implemented in real time and modeling and simulation of dynamic systems, with emphasis in the study and controllers' of position of joints robotics. Initially the study of the constituent elements of a joint robotics will be accomplished, such as: DC motor, inertia, reducers and joining. To leave of that study it will be possible the definition of the control strategy to be used, including the development of a generator of trajectories used inside as reference of a control mesh involving these elements.*

Keywords: *Kinematic, modeling, dynamical.*

1. INTRODUCTION

Actually, the number of robots installed in the industry increases progressively because of its capacity to realize operations that demands flexibility, quickness and precision.

In most industrial applications the robot tasks are programmed by learning without the need of a geometrical model. By this way, its trajectory is defined through a set of angles associated to the angular movement of each degree of freedom of the robot, which after interpolation by an algorithm, will act as reference signal for positioning controllers located at each joint and that compare the signals deriving from the position transductors of the joints.

For many operations the operator defines the tasks or reference movements of the controller with respect to a coordinate system that is solidary to the end effector of the robot (in the Cartesian space), Fig. 1. By this way, the desired movements (expressed in angular coordinates) and the control laws are in different coordinate systems, demanding the implementation of fast algorithms for the inversion of the geometrical model, for generation of the reference trajectory in angular coordinates.

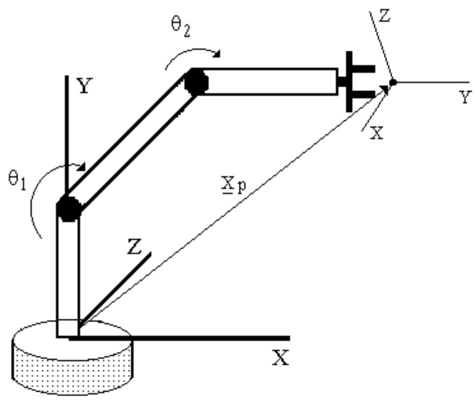

Figure 1: Coordinate system.

This reference trajectory after having compared with the final position , Fig. 2,of the load, would generate a error (ε) that it would be minimized by the controller, through an algorithm implemented in a microprocessor involving the theory of nonlinear control.

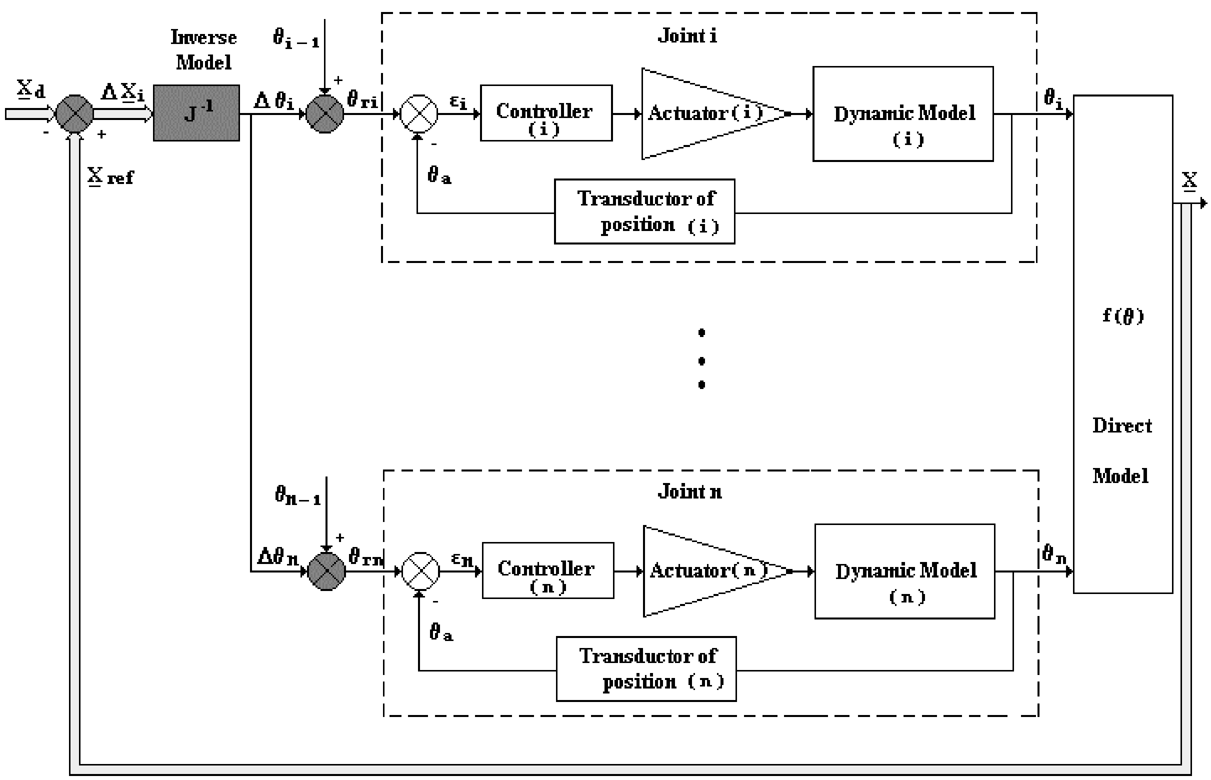

Figure 2: Inverse kinematic model.

where J^{-1} is the Jacobian matrix inverse;
$f(\theta)$ is the Kinematic direct model.

The section 1 focus the implementation of numerical algorithms for the solution of the kinematic inverse problem, in the section 2 the modeling and simulation of dynamic systems was developed, with emphasis in the study and controllers of position of joints robotics.

2. SECTION 1

2.1 Geometrical model and kinematic inverse problem

The geometrical model of a robot expresses the position and orientation of its end effector with respect to a coordinate system solidary to the base of the robot, in function of its generalized coordinates (angular coordinates in the case of rotational joints). The geometrical model is represented by the following expression:

$$\underline{x} = f(\underline{\theta})$$

where $\underline{\theta} = (\theta_1, \theta_2, ..., \theta_n)$:angular position vectors for the joints;
$\underline{x} = (X, Y, Z, \psi, \theta, \phi)$: position vector, where the three first terms denote the cartesian position and the three last terms stand for the orientation of the end effector.

This relation may be expressed mathematically by a matrix that relates the system of coordinates solidary to base of the robot with a system of coordinates associated to its end effector. This matrix is called homogeneous passage matrix and is obtained from the product of the homogeneous transformations matrix, $A_{i\ i-1}$, that relates the system of coordinates of an element *i* with the system of the previous element *i*-1, that is

$$T_n = A_{0.1} * A_{1,2} * .... * A_{n-1,n} = [\ \underline{n}\ \underline{s}\ \underline{a}\ \underline{p}\ ]$$

where $\underline{p} = [\ p_x , p_y , p_z\ ]$: position vector;
$\underline{n} = [\ n_x\ n_y\ n_z\ ]$, $\underline{s} = [\ s_x\ s_y\ s_z\ ]$ and $\underline{a} = [\ a_x\ a_y\ a_z\ ]$: orthonormal vector that describes the orientation.

The description of the transformation matrix is done through the usage of the Denavit-Hartenberg procedure, after the obtention of the four parameters θ_i, .a_i, d_i and α_i.

The need for finding references in angular coordinates referring to the tasks defined in the Cartesian space is expressed mathematically by the inversion of the geometrical model, that is:

$$\underline{\theta} = f^{-1} (\underline{x})$$

Through the function *f* it is possible to calculate the movement the end effector resulting from the movement of the joints. This function is nonlinear and has no nontrivial analytical solution.

2.2 Pseudoinverse matrix

In many cases there is a solution for a system of linear equations even if there is no associated inverse matrix. This problem and many others may be solved through the usage of the pseudoinverse matrix (A^+), also called generalized inverse matrix. The solution for the generalized inverse of a given matrix must obey the following properties (Nashed, 1976):

- to be reduced to A^{-1} in the case that A is not singular;
- to exist always;
- to possess some of the properties of the inverse matrix (or modifications of these);
- when used in the place of the inverse matrix, to be able to provide sensible responses for important questions as: equations consistence or least-square solutions.

Moore and Penrose (Huang, 1983) defined the principle of the pseudoinverse, A^+, as the unique solution to the following set of equations:

$$A \times A^+ \times A = A$$
$$A^+ \times A \times A^+ = A^+$$
$$(A \times A^+)^t = A \times A^+$$
$$(A \times A^+)^t = A^+ \times A$$

2.3 Algorithm

The development of a numerical algorithm (Sá, 1996) to find the angular positions for a task defined with respect of its end effector in the Cartesian space, contains the solution of the inverse kinematic problem through the usage of a recursive numerical method that uses the calculation of the kinematic model and of the Jacobian inverse matrix for the manipulator. The Greville's method is used for the calculation of the pseudoinverse matrix of the Jacobian matrix.

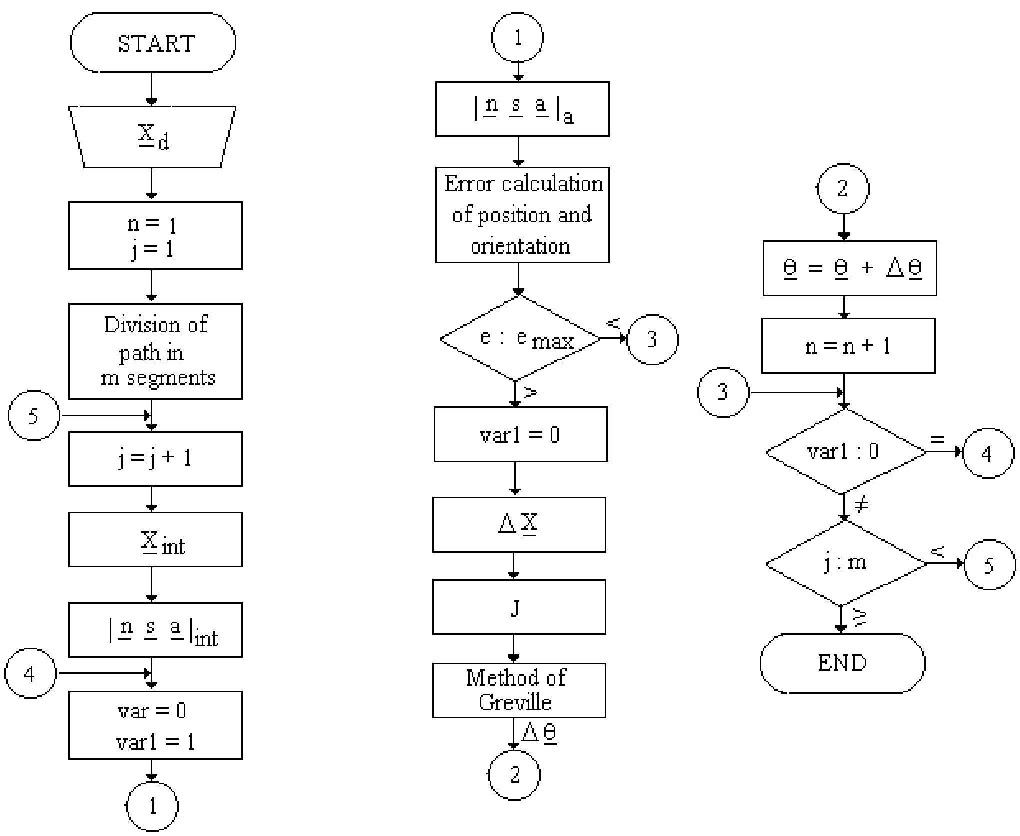

Figure 3: Algorithm for the kinematic inverse problem.

With the aim of validating the algorithm, shown in Fig. 3 different simulations were made assessing the behavior of the trajectory (angular space). For this purpose it was used the geometrical model of the submarine manipulator Kraft, with 6 rotational joints. Moreover, the trajectory followed by the end effector (Cartesian space) of the manipulator was plotted for all simulations.

The plot presented in this paper, Fig. 4 and 5, show the trajectory obtained for the manipulator moving from an initial position (in mm) and orientation (in degrees), $\underline{x}_i$, (776.9,0,933.1,0,90,0), that correspond to the angular configuration, in degrees, (0, 90, -90, 0, 90, 0), to a final target, $\underline{x}_d$, (776.9, 0, 933.1, 25, 63, 75). The final angular configuration achieved, in degrees, is (-11.05, 37.81, -139.67, 131.42, 113.03, 167.51).

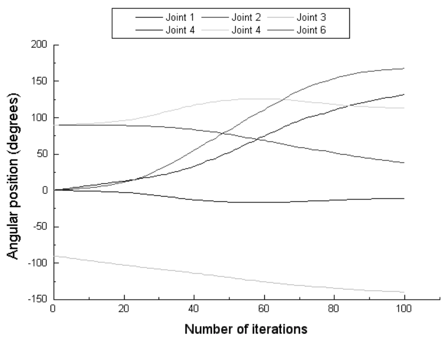

Figure 4: Angular evolutions.

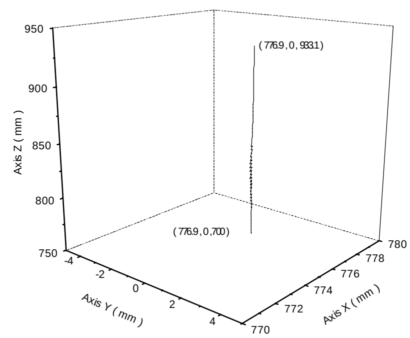

Figure 5: Trajectory of end effector.

3. SECTION 2

3.1 Modeling and dynamics control

After the solution of trajectory problem the complete study of dynamical and control system was realized. The control of system was realized for each joint (desacoupled joint).

The signs of reference obtained in angular coordinates by interpolator of trajectories starting from the comparison of these reference signs with the coming angular positions of the transductors of position of each joint (encoder incremental) the controller will make the due corrections being taken into account the robot's dynamic model in study.

A diagram of blocks for the nonlinear case is shown in the Fig. 6, in him we presented the structure of the System (model + controls).

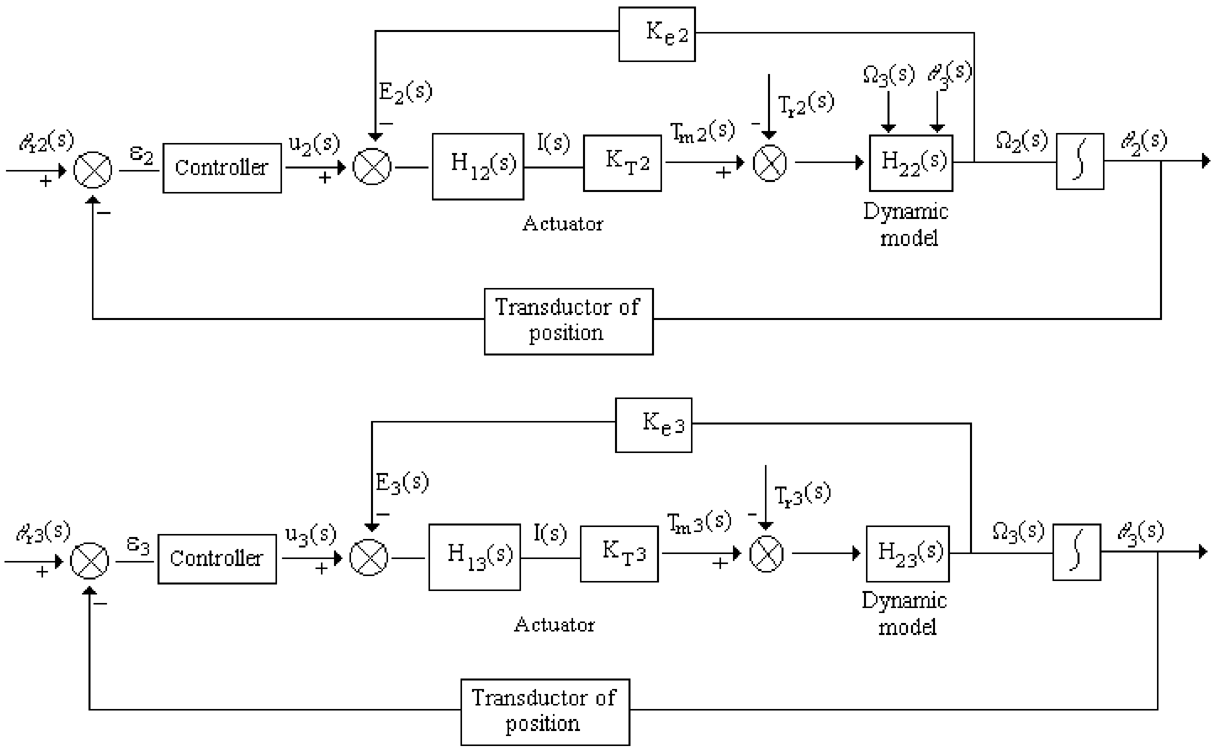

Figure 6: Diagram of blocks for the nonlinear equations

Initially it is presented the equations that govern the electric motor, increasing to follow the load with its joints. Starting from the simulations of the electric motor with its joints, a reference trajectory is generated for the mesh of control of the system in subject.

3.2 Actuator model

In this work a DC motor will be used, traditionally used in industrial robots. The same analysis type can be accomplished for another action types.
The electric equations, mechanics and of joints they are presented below:

$v = L\dot{i} + Ri + K_v\dot{\theta}$ electric equation

$T = J_m\ddot{\theta} + C_m\dot{\theta}$ mechanical equation

$T = K_T i$ joints equation

where:
i(t) - current (A);
R - induced resistance (W)
L - inductance (H);
u - tension applied in the circuit of the armor (V);
Jm - moment of inertia of the motor (kg m^2).
Ke - constant of the force against-eletromotriz (V/rad s^{-1});
K_T - constant of torque ((Nm/A);
Tr - resistant torque due to losses (Nm);
Tm - mechanical torque (Nm).

The equations above can be represented by the following diagram of blocks, Fig. 7, making the use of the formulation of Laplace.

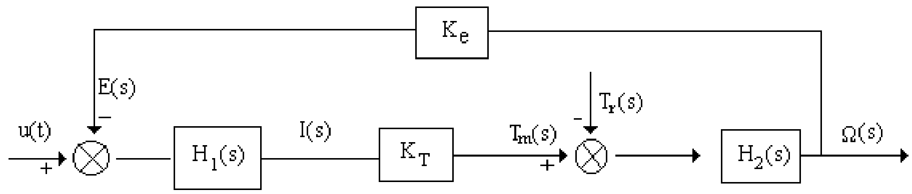

Figure 7: Diagram of Block of the DC Motor

3.3 Dynamic model of manipulator

The manipulator dynamic behavior can be described by a group of differential equations called dynamic equations of motion.

For a rigid manipulator with two degrees of freedom the equations are:

$$J_1 \ddot{\Theta}_1 + F_1 \dot{\Theta}_1 + \Gamma_1 = \tau_1$$

$$J_2 \ddot{\Theta}_2 + F_2 \dot{\Theta}_2 + \Gamma_2 = \tau_2$$

3.4 System of control and results

The control of a system can be defined as a system whose the proposition is calibrate or to adjust the flow of energy in a wanted way. A system of control mesh in shut uses the signs of the exit to modify the actions of the system with the aim of reaching the specified objective. Starting from a reference sign input that compared the sign output of the system generates an error that with the an element controller's performance, this signs correspondent after having amplified are send to system action.

In this work we used a controller of the type PID including in the mesh of control of the system in subject, Fig. 8.

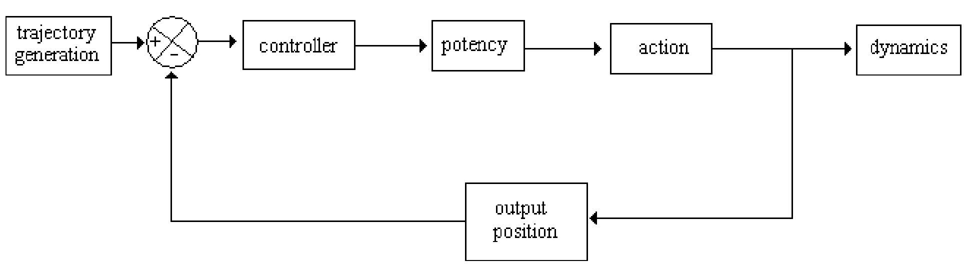

Figure 8: Basic diagram of blocks of a system of Control

The relevant data for the simulation are the following:

R = 3,0 ohms;
L = 0.005 henry
Jm = 1.412e-4 Kg m^2
Cm = 2.7e-4 Nm/rad s^{-1}
K_T = 0.001 Nm
g = 9.8 m/s^2

The reference sign input used in this work was built being taken into account the constant of time of the system and the speed of the actioner. Controller PID's gains are 10, 5 and 2 respectively. This reference sign input is then presented in Fig. 9. The answer of the control of the motor with reduction and load is presented in Fig. 10.

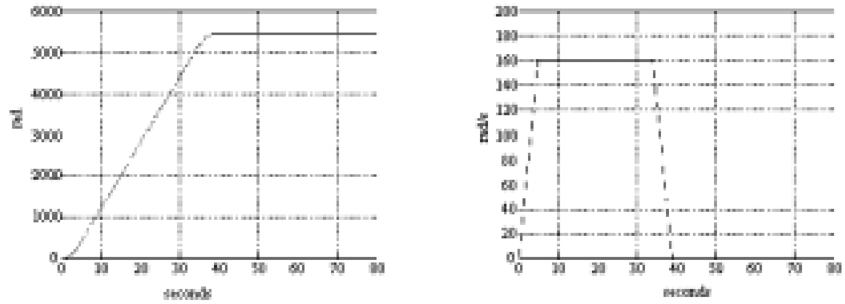

Figure 9: Reference sign input for the control system.

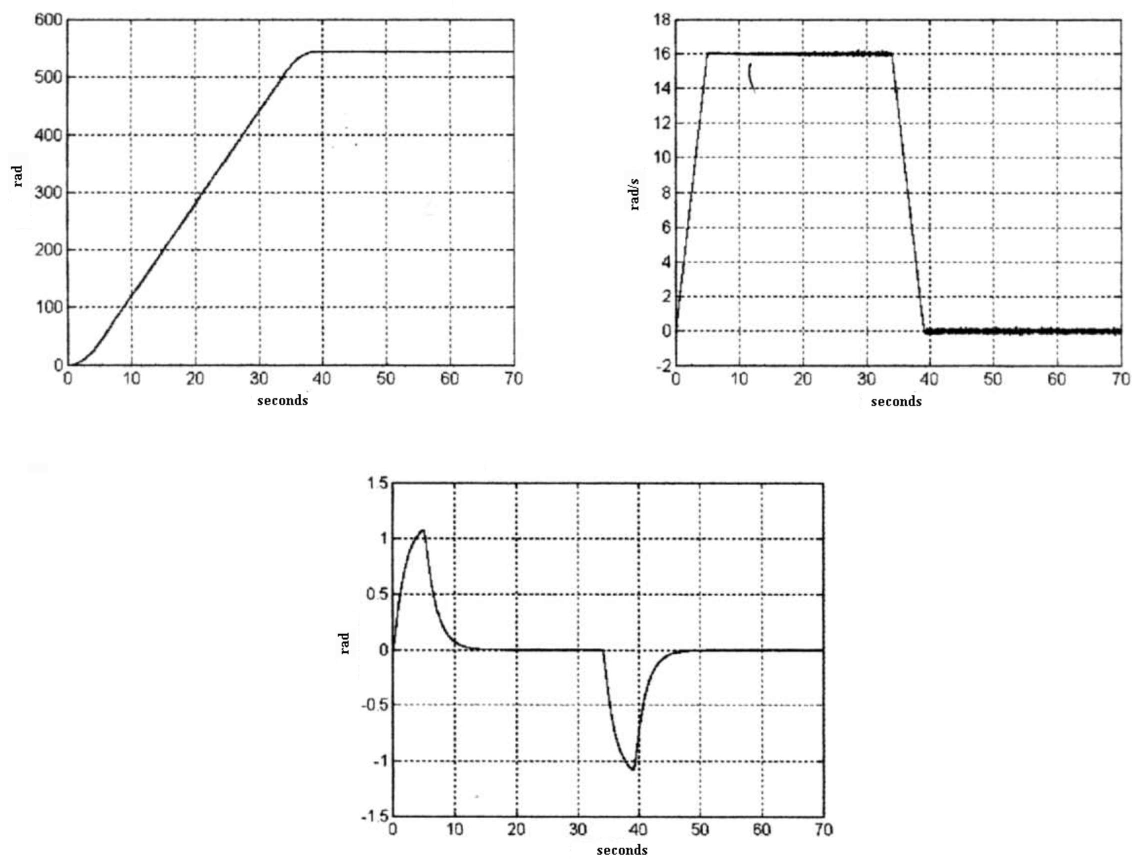

Figure 10: Answer of the control of the motor with reduction and load.

4. Conclusions

The generation of trajectories through the usage of the kinematic inverse model presented excellent results and the computational simplicity of the method allows the implementation of anti-collision strategy algorithms.

In this work a systematic one was proposed for mathematical modeling. Initially the dynamic modeling of the different elements of a robotic system, and a system was presented for view and control that it is being implemented in laboratory.

Through simulations, using the parameters of the system, the viability of the project was verified and with supported experimental, described previously, it will be possible the validation of the obtained data.

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